A New Model for the Apparent Characteristic Impedance of Finned Waveguide and Finlines

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Abstract **—This paper presents a new model for the apparent characteristic impedance of finned waveguide and finlines from the standpoint of per-unit-length capacitance of the fins, obtained by conformal mapping, and of the waveguide, obtained from cutoff wavelengths, without going into the controversy of definition. The model strongly supports the power–voltage definition. Closed-form synthesis equations have also been derived which are within 1.5 percent of the analysis equations. The model will be useful in computer-aided design of millimeter-wave finned waveguide and finlines.**

I. Introduction

THE CONCEPT OF IMPEDANCE is indispensable in microwave circuit design since the distribution of microwave energy among the elements, through the interconnections, is decided by the impedance. Therefore, the exact prediction of circuit performance depends upon how accurately the impedance of its elements is known.

Microwave circuits are made of distributed networks using pieces of transmission lines which are characterized in terms of their phase constants and characteristic impedances.

As discussed by Getsinger [1] for microstrip line, we can also define the "apparent" characteristic impedance of finned waveguide or finline. This describes how finned waveguide or finline exchanges power with a TEM line or a two-terminal device, just as the characteristic impedance is the parameter which determines how a TEM line exchanges power with another transmission line or with a two-terminal device.

The definition of characteristic impedance is unique in the case of a purely TEM line. The controversy regarding the definition of characteristic impedance arises when the transmission line supports a non-TEM mode. The problem of the apparent characteristic impedance of quasi-TEM line (like microstrip) has been solved by experimental modeling [1] and by theoretical analysis [2]. For purely non-TEM planar transmission lines such as finned waveguide and finline, the definition of the apparent characteristic impedance still remains an open question. Till now there have been no adequately conclusive experimental results or theoretical arguments in favor of any of the three possible definitions, i.e., voltage/current, power/voltage and power/current. Moreover, neither the voltage nor the current can be uniquely defined in a finned waveguide or in a finline. However, while the experimental findings of Meinel and Rembold [3] tend towards the voltage/current definition, those of Willing and Speilman [4] are in favor of the power/voltage definition. In the above definition, voltage is defined as the line integral of the electric field over the shortest path between the fins, and the current is the total longitudinal current flowing in the line. Still, the experimental findings show a wide divergence from the numerical computations [5], [6]. Approximate ridge guide definitions of Meier [7] differ by 8 to 9 percent from numerically computed values [5], [6]. On the other hand, the slow rise in the characteristic impedance with increasing frequency at the upper edge of the usable band, as predicted by numerical computation, has not been experimentally confirmed. Keeping all this in mind, the present work derives the characteristic impedance from the per-unit-length capacitance standpoint.

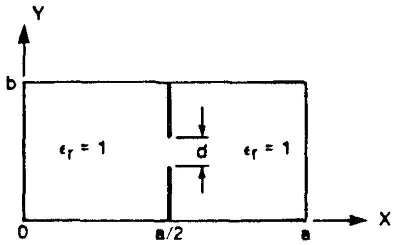

Fig. 1. Finned waveguide.

II. Theory

A. Analysis of Finned Waveguide

The finned waveguide in Fig. 1 can be considered to be a combination of a finned parallel plate of infinite width, as shown in Fig. 2(a), and a rectangular waveguide, as shown in Fig. 2(b).

The per-unit-length capacitance between the fins of the parallel-plate waveguide is uniquely defined and is obtained accurately from conformal mapping as [8]

$$C_f = \left(\frac{2}{\pi}\right) \ln \csc\left(\frac{\pi}{2}\frac{d}{b}\right). \quad (1)$$

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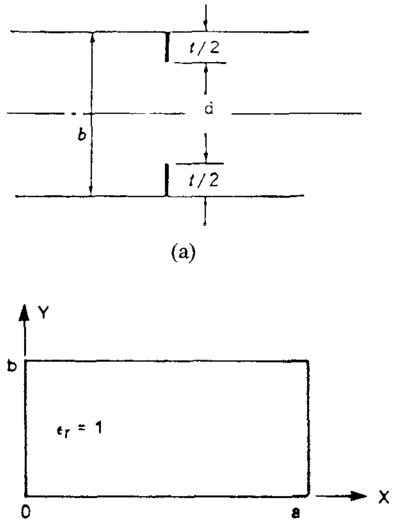

Fig. 2. (a) Finned parallel plate. (b) Rectangular waveguide.

Now, if the capacitance between the midpoints of the broadwalls of the waveguide is C_w, then it follows from the definition of characteristic impedance [9], that the characteristic impedance of the finned waveguide at frequency f will be given by

$$Z_0(f) = \frac{Z_{0\infty}}{\sqrt{1-(\lambda/\lambda_{ca})^2}} \tag{2}$$

where λ is the wavelength corresponding to frequency f, and λ_{ca} is the cutoff wavelength of the finned waveguide. $Z_{0\infty}$, the characteristic impedance at infinite frequency, is given by [9]

$$Z_{0\infty} = \frac{\eta_0}{C_w + C_f} \text{ ohms} \tag{3}$$

where $\eta_0 = 120\pi\ \Omega$ (free-space impedance).

Since the characteristic impedance of the rectangular waveguide $Z_{0\infty}$ cannot be defined uniquely, C_w can have three different values depending upon the definition. For example, the power–voltage definition gives, (TE_{10} mode)

$$Z_{0\infty w}^{PV} = 2\eta_0(b/a) \text{ ohms}. \tag{4}$$

The voltage–current definition gives

$$Z_{0\infty w}^{VI} = \left(\frac{\pi}{2}\right)\eta_0(b/a) \text{ ohms} \tag{5}$$

while the power–current definition gives

$$Z_{0\infty w}^{PI} = \left(\frac{\pi^2}{8}\right)\eta_0(b/a) \text{ ohms}. \tag{6}$$

Corresponding to each definition of $Z_{0\infty w}$, a C_w can be defined. The question now arises which one should be used in (3) to define $Z_{0\infty}$. Although the characteristic impedance could not be defined uniquely, the propagation constant of the finned waveguide can be defined uniquely. Or, in other words, the cutoff wavelength can be defined uniquely. Using the perturbation theory, the cutoff wavelength of the finned waveguide can be written as

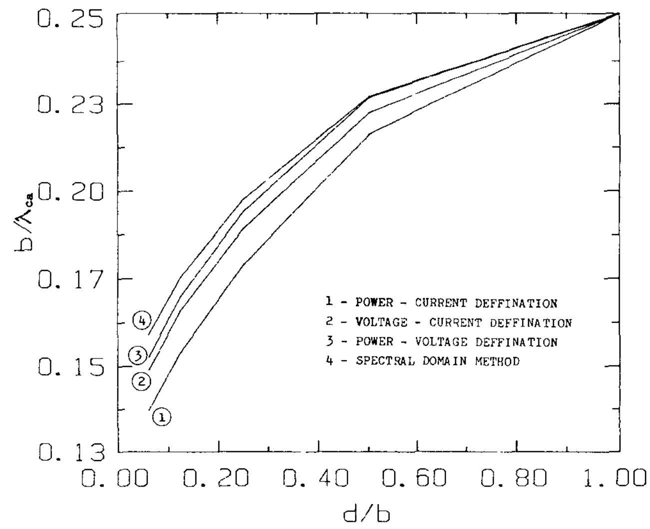

Fig. 3. Comparison of cutoff wavelengths.

$$\lambda_{ca} = 2a\left[1 + \frac{\Delta C}{C_w}\right]^{1/2} \tag{7}$$

where

$$\Delta C = C_f \tag{8}$$

and

$$C_w = C_{wPV} = \frac{1}{2}\frac{a}{b} \quad \text{(from (4))} \tag{9}$$

$$= C_{wVI} = \frac{2}{\pi}\frac{a}{b} \quad \text{(from (5))} \tag{10}$$

$$= C_{wPI} = \frac{8}{\pi^2}\frac{a}{b} \quad \text{(from (6))}. \tag{11}$$

Let us compute λ_{ca} using all three of the above definitions of C_w and compare the results with accurately computed values of λ_{ca} from the spectral-domain method [10]. Fig. 3 shows that the best results are obtained by using the power–voltage definition or $C_w = C_{wPV}$. The difference between the perturbational formula and the spectral-domain results increases for smaller d/b values because the field distribution, in the waveguide cross section, tends to shift from the sinusoidal form as d/b decreases. Consequently, (9) needs to be modified to

$$C_{wPV} = \left(\frac{1}{G}\right)\left(\frac{1}{2}\right)\left(\frac{a}{b}\right). \tag{12}$$

The factor G has been determined empirically to be

$$G = 1 + (0.129\sqrt{x} + 0.00915)\sqrt{\frac{b}{a}} \tag{13}$$

where

$$x = \ln\csc\left(\frac{\pi}{2}\frac{d}{b}\right). \tag{14}$$

Equation (7) together with (12), (13), and (14) gives λ_{ca} within 0.4 percent of spectral-domain results.

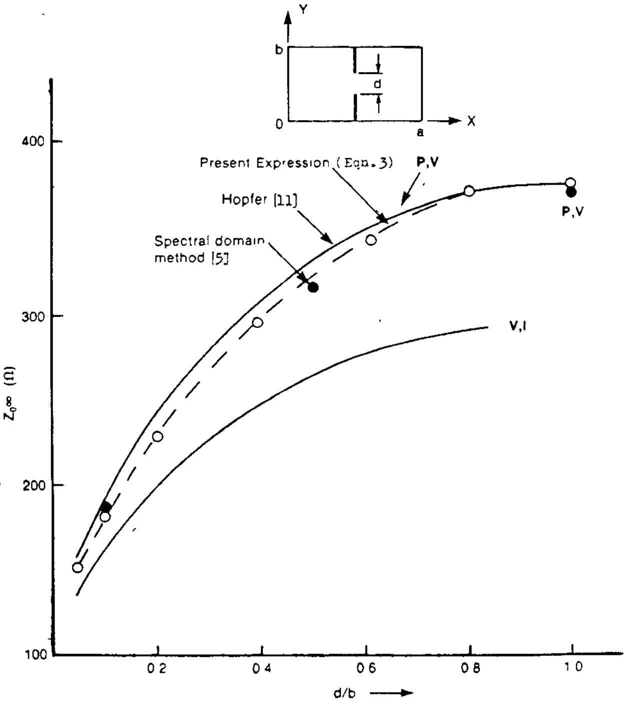

Fig. 4. Comparison of present model with ridged guide definition and spectral-domain results.

Therefore, using (1), (2), (3) and (12), (13), (14), the characteristic impedance of the finned waveguide in Fig. 1 is given by

$$Z_0(f) = \frac{2\eta_0(b/a)G}{\left[1+\frac{4}{\pi}\left(\frac{b}{a}\right)Gx\right]\sqrt{1-(\lambda/\lambda_{ca})^2}}. \quad (15)$$

Fig. 4 compares results obtained by using (15) with those obtained from Hopfer's [11] ridged waveguide model and the spectral-domain method. A close agreement among the results is observed. The results agree exactly for $d/b = 1$.

B. Synthesis of Finned Waveguide

The factor G appearing in (15) can be approximated within 1 percent in the numerator as

$$G = 1 + (r_1 x + r_2)\sqrt{b/a} \quad (16a)$$

where

$$(r_1, r_2) = (0.16814, 0.00894), \quad \text{for } d/b \geqslant 1/3 \quad (16b)$$

$$= (0.051496, 0.08739), \quad \text{for } d/b \leqslant 1/3 \quad (16c)$$

and in the denominator as

$$G = 1 + 0.2\sqrt{b/a}. \quad (16d)$$

Then, solving (15) for x gives

$$x = \frac{1}{2}\left\{-p + (p^2 - 4q)^{1/2}\right\} \quad (17)$$

$$p = \left[N\left\{2 - \frac{1}{4}(b/a)^2(\lambda/b)^2\right\}Z_0^2(f) - 2\alpha_1\alpha_2\right] / \left[N^2 Z_0^2(f) - \alpha_1^2\right] \quad (18a)$$

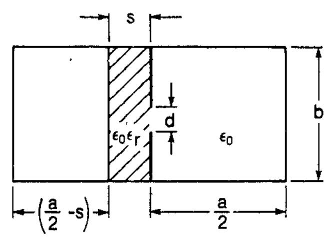

Fig. 5. Unilateral finline.

and

$$q = \left[Z_0^2(f)\left\{1 - \frac{1}{4}(b/a)^2(\lambda/b)^2\right\} - \alpha_2^2\right] / \left[N^2 Z_0^2(f) - \alpha_1^2\right] \quad (18b)$$

where

$$N = \frac{4}{\pi}\left(\frac{b}{a}\right)\left(1 + 0.2\sqrt{b/a}\right) \quad (19)$$

$$\alpha_1 = 2\eta_0 r_1 (b/a)^{3/2} \quad (20a)$$

$$\alpha_2 = 2\eta_0 (b/a)\left(1 + r_2\sqrt{b/a}\right). \quad (20b)$$

The above synthesis equation (17) gives results within 1.5 percent of the analysis equation. Therefore, an accurate synthesis is possible by improving the results of the synthesis equation using the analysis equation.

III. Application to Unilateral Finline

A. Analysis of Unilateral Finline

The effective dielectric constant in unilateral finline shown in Fig. 5 can be written as

$$\epsilon_e(f) = k_e - \left(\frac{\lambda}{\lambda_{ca}}\right)^2 \quad (21)$$

where k_e, the equivalent dielectric constant of the finline, can be obtained from [12] and [13], and λ_{ca} is given by (7) together with (1) and (12) through (14). Therefore, the frequency-dependent characteristic impedance of unilateral finline is given by

$$Z_0(f) = \frac{2\eta_0(b/a)G}{\left[1+\frac{4}{\pi}(b/a)Gx\right]\sqrt{k_e-(\lambda/\lambda_{ca})^2}}. \quad (22)$$

Equation (22), however, does not necessarily fully agree with numerically computed $Z_0(f)$ [5] based on the power–voltage definition, as shown in Figs. 6 and 7. It has been found that the numerically computed $Z_0(f)_{PV}$ can be accurately modeled as (see Figs. 6 and 7)

$$Z_0(f)_{PV} = Z_0(f)\left\{\frac{1-1/k_e}{1-1/k_c}\right\}^{1/2}\left(\frac{k_e}{k_c}\right) \quad (23)$$

where $Z_0(f)$ is given by (22), and k_c is the value of k_e at cutoff.

Equation (23) shows a rise in $Z_0(f)_{PV}$ with frequency. However, the rise is slow and significant only outside the useful frequency band.

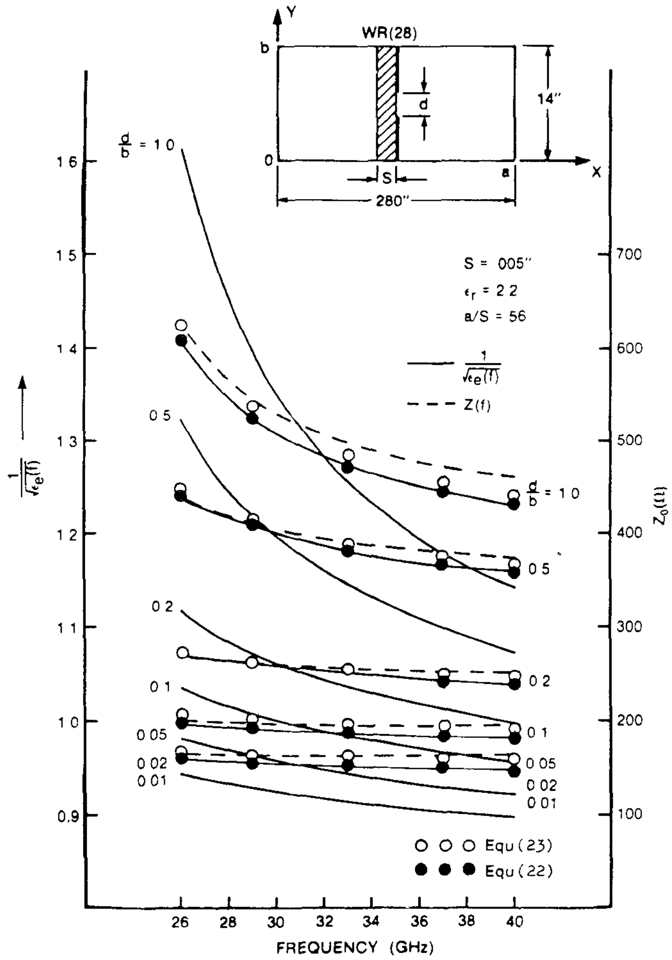

Fig. 6. Comparison with spectral-domain results.

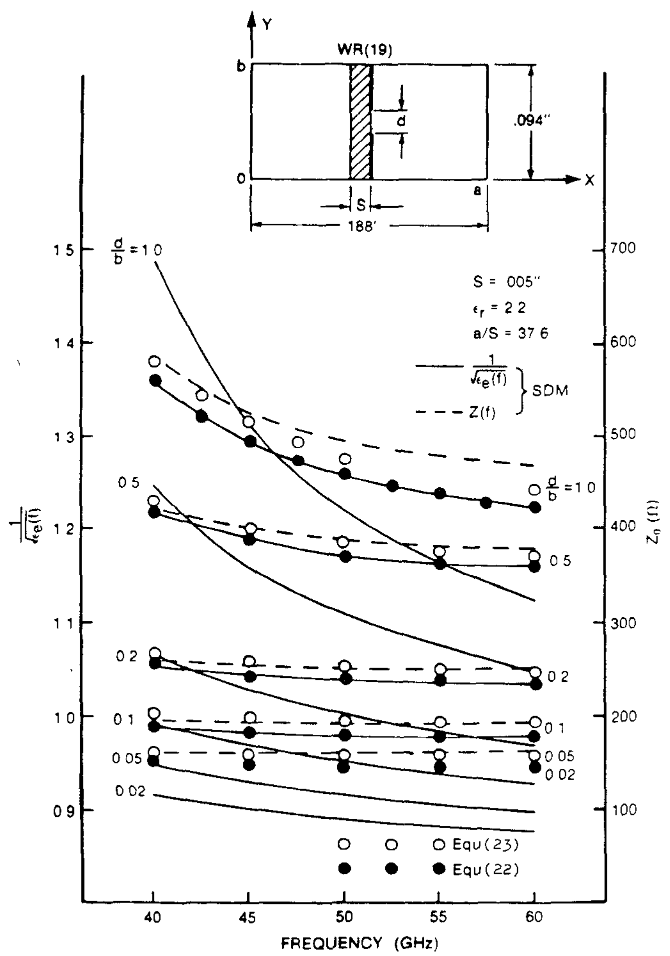

Fig. 7. Comparison with spectral-domain results.

B. Synthesis of Unilateral Finline

For most practical finline applications $\epsilon_r < 2.5$ and $s/a \ll 1$ (see Fig. 5). Therefore, one can make a first-order approximation by considering the equivalent dielectric constant to be frequency-independent and given by [12] its value k_c at the cutoff frequency as

$$k_e = k_c = 1 + \frac{s}{a}(a_1 x + b_1)(\epsilon_r - 1) \tag{24}$$

where

$$a_1 = [0.6177 \ln(a/s) - 0.5457]^2$$
$$b_1 = 2.42 \sin[0.556 \ln(a/s)].$$

Substituting (24) into (22) gives the following cubic equation for x:

$$x^3 + px^2 + qx + r = 0 \tag{25}$$

where

$$p = [AN^2 + 2BN - \alpha_1^2]/BN^2$$
$$q = \left[B + AN - \frac{1}{4}(b/a)^2(\lambda_b/b)^2 - 2\alpha_1\alpha_2\right]/BN^2$$
$$r = \left[A - \frac{1}{4}(b/a)^2(\lambda_b/b)^2 - \alpha_2^2\right]/BN^2$$
$$A = 1 + b_1(s/a)(\epsilon_r - 1)$$
$$B = a_1(s/a)(\epsilon_r - 1)$$

and α_1 and α_2 are given by (20a) and (20b), respectively.

The values of x obtained from (25), when substituted into (22), yield results within 2 percent of the design value. The accuracy can be improved to within 1 percent by using the analysis equation iteratively with the x obtained from (25) as the starting value. Such an iterative technique can also be used for $\epsilon_r \not< 2.5$ and $s/a \not\ll 1$. One can also use (23) in such an iterative synthesis technique if one has more confidence in the power–voltage definition of the characteristic impedance. A general computer program has been developed to implement the above technique and is being used as a CAD tool [14]. The present modeling can also be used for the insulated finline using closed-form equations [15].

A similar model has been developed for bilateral finline and will be reported soon.

IV. Application to Single-Ridge Finline

The model derived herein for the analysis and synthesis of finned waveguide and finline can also be applied to a single-ridge finline with the following interpretation: b is twice the height of the housing of the single-ridge finline, and d is twice the gap between the tip of the fin and the broadwall of the housing. The interpretation remains the same for the characteristic impedance; however, the characteristic impedance of the single-ridge finline is half that of the finline.

V. Conclusions

This paper defines the characteristic impedance of finned waveguides and finlines from the per-unit-length capacitance standpoint, without going into the controversy of definition used in numerical analysis of the structures. The total per-unit-length capacitance of the fins is obtained from the results of conformal mapping. The new definition strongly supports the power–voltage definition. Simple synthesis equations have been derived under practical assumptions. The model throws new light on the controversial definition of the apparent characteristic impedance of finline and will be vital to the CAD of millimeter-wave circuits.

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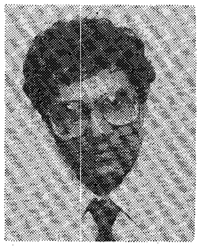

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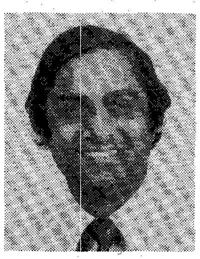

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